15. (a) Prove that the equation $\frac{ah}{\cos\theta} - \frac{bk}{\sin\theta} = a^2 - b^2$ has four roots and that the sum of the four values of θ which satisfy and its is equal to an odd multiple of π radians.

Or

(b) Express $\frac{\sin b\theta}{\sin\theta}$ in terms of $\cos\theta$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that $\sum_{0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.

17. Find all the rational roots of the equation $4x^3 + 20x^2 - 23x + 6 = 0$ using Newton's method.

18. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ and hence determine its inverse.

19. Solve the following systems of linear equations by Cramer's rule. $x + 2y + 3z = 6$, $2x + 4y + z = 7$ $3x + 2y + 9z = 14$.

20. Expand $\sin^4\theta \cos^2\theta$ in a series of cosines of multiples of θ.

NOVEMBER/DECEMBER 2023

FAEL12/CAEL12 — BASIC MATHEMATICS - I

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

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1. Prove that the Co-efficient of x^n in the expansion of $1 + \frac{1+2x}{1!} + \frac{(1+2x)^2}{2!} + \frac{(1+2x)^3}{3!} + \ldots$ to ∞ is $\frac{2^n e}{n!}$.

2. Write the expansion of log $(1 + x)$.

3. Diminish the roots of the equation $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

4. Define reciprocal equation.

5. Define skew symmetric matrix.

6. Find the eigen value of the matrix $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.

7. If $A = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$ then find A^{-1}.

8. What is singular matrix?

9. Write the expansion of $\tan n\theta$.

10. Write the co-efficient of $\cos^n\theta$ is the expansion of $\cos n\theta$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Find the sum to infinity of the series $1+\frac{3}{4}+\frac{3\cdot 5}{4\cdot 8}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12}+\ldots\ldots$

Or

(b) If a, b, c denote these consecutive integers, show that $\log_e b=\frac{1}{2}\log_e a+\frac{1}{2}\log_e c+\frac{1}{2ac+1}+\frac{1}{3}\frac{1}{(2ac+1)^3}+\ldots.$

12. (a) Remove the fractional co-efficients from the equation $x^3+\frac{1}{4}x^2-\frac{1}{16}x+\frac{1}{72}=0$.

Or

(b) Solve the equation $6x^5-x^4-43x^3+43x^2+x-6=0$.

13. (a) Prove that the matrix is orthogonal $\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$.

Or

(b) Find A satisfying the matrix equation $\begin{bmatrix}2&1\\3&2\end{bmatrix}A\begin{bmatrix}-3&2\\5&-3\end{bmatrix}=\begin{bmatrix}-2&4\\3&-1\end{bmatrix}$.

14. (a) Evaluate the determinant $\begin{bmatrix}a&h&g\\h&b&f\\g&f&c\end{bmatrix}$.

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Or

(b) Find $3A-2B+C$ if $A=\begin{bmatrix}3&3&3\\3&2&-2\\2&1&5\end{bmatrix}$ $B=\begin{bmatrix}4&4&4\\4&1&6\\7&10&0\end{bmatrix}$ and $C=\begin{bmatrix}5&5&5\\6&0&1\\8&9&0\end{bmatrix}$.